Figure 5 compares the L/D ratio for a flat plate ($\beta = 0$) with the results of Mirels and Lewellen.[2] Agreement is good for small values of the angle of attack.

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Structure of Turbulent Diffusion Flames

PAUL M. CHUNG*
University of Illinois, Chicago, Ill.

Nomenclature

C_p	=	constant pressure specific heat
h	=	$(C_p/\Delta h^0)t$
Δh^0	=	heat of combustion
L	=	width of the shear or mixing layer
$\hat{M}$	=	molecular weight
m	=	$(d\hat{M}_p/a\hat{M}_r)z_r$
n	=	$(d\hat{M}_p/b\hat{M}_f)z_f$
t	=	instantaneous absolute temperature
U_k	=	$u_k - \langle u_k \rangle$
u_k	=	absolute velocity vector in tensor notation
u,v,w	=	x, y, and z components of velocity, respectively
x,y	=	Cartesian coordinates
x_k	=	position vector in tensor notation
Y	=	y/L
z	=	Cartesian coordinate, or the instantaneous mass fraction
λ	=	integral scale in the limit of small mean velocity gradient
ρ	=	density
$\langle Q \rangle$	=	$\int f Q d\mathbf{U}$

Introduction

IN the existing theories wherein the turbulent transport is described phenomenologically as a function of the local properties in analogous to the laminar transport, the mixing of the two fluid elements containing two different chemical reactants will immediately allow the reaction to commence between the two reactants. Therefore, these turbulent mixing theories predict the existence of an infinitesimally thin diffusion flame sheet in the chemically equilibrium limit, as it is with the laminar diffusion flame. A typical analysis leading to such a flame sheet was given by Libby.[1] A chemical reaction, however, is a molecular process, and the mixing of the fluid elements is not sufficient for the combustion of the initially unmixed reactants.

The molecular diffusion of chemical species between the fluid elements which have been mixed takes finite amounts of time and, therefore, the combustion zone must be of finite thickness even in the limit of an infinitely large Damkohler number.

The existing experimental results[2–4] of turbulent diffusion flames bear out the aforementioned aspect of the combustion. The mean concentration profiles of the reactants show that the flame zone wherein both reactants coexist is very thick indeed.

In order to rectify the aforementioned difficulties with the conventional phenomenological theories in describing the mixing and chemical reaction, a simplified statistical theory[5,6] has been developed by the present author for the high turbulence-Reynolds number flows. This theory has been successfully employed to analyze the turbulent flowfield with a uniform mean velocity gradient,[5] which we define as the homologous flowfield. In the present paper, this theory is employed to analyze the structure of the diffusion flame established within an homologous turbulent shear field. The homologous field is chosen because it is the simplest shear field for analysis, and because it is expected that the basic structure of the flame is independent of the flow configuration and boundary conditions as it is with the laminar flames.[7] The details of the results and analyses to be discussed here are found in the AIAA preprint.[8]

Analysis

We consider the mixing layer formed between the two isotropic streams of equal turbulence energy E shown in Fig. 3 of Ref. 5. In order that the flow be homologous, we assume that the mixing layer is bound by two slippery planes, respectively at $Y = 0$ and $Y = 1$, which are perfectly pervious to mass and momentum. We further consider that the stream at $Y = 0$ consists of a pure fuel whereas the stream at $Y = 1$ consists of an oxidant and a chemically inert gas. The two streams are considered to be at the uniform temperatures of $t_1(0)$ and $t_2(1)$, respectively.

We consider that the mean fluctuations of all scalar quantities are zero in the bounding streams. The fluctuations of the scalar quantities in the mixing layer are then caused by the mixing of the chemical species and by the chemical reaction taking place in the layer.

The following one step chemical reaction is considered for the combustion:

$$a(\text{fuel}) + b(\text{oxidant}) \xrightarrow{k} d(\text{product})$$

where a, b, and d denote the number of moles.

The specific rate coefficient k is considered to be given by

$$k = k_o \exp[-\Delta E/(Rt)] \tag{1}$$

where k_o is a constant and ΔE and R are the activation energy and the gas constant, respectively.

The instantaneous rate of generation of the combustion product, W_p, is then given by the law of mass action[7] as

$$W_p = K[\exp - (\Delta E/R)(1/t - 1/t^*)]z_r^a z_f^b \tag{2}$$

where

$$K = k_o(d\hat{M}_p/\hat{M}_r^a\hat{M}_f^b)\rho^{a+b-1} \exp - \Delta E/(Rt^*) \tag{3}$$

In the preceding equations, $(\,)^*$ represents the mean chemical equilibrium value at one of the flame edges to be discussed subsequently. The subscripts r, f, c, and p denote the fuel, oxidant, inert gas, and the product, respectively. Note that the meanings of r and f were interchanged in Ref. 8 by mistake.

The instantaneous rates of generation of the fuel, W_r, the oxidant, W_f, and the temperature, W_t, by chemical reaction can be readily related to that of the combustion product.[7]

The starting point of the analysis is the Fokker-Planck equations developed earlier[5] for the flow and scalar fields with no laminar sublayers and mean pressure gradients

$$u_j \frac{\partial f}{\partial x_j} = \frac{\langle U_k U_k \rangle^{1/2}}{2\lambda}\left(1 + \frac{L}{\langle u \rangle_\infty}\frac{\partial \langle u \rangle}{\partial y}\right) \times \left[2\frac{\partial}{\partial U_j}(fU_j) + \frac{\langle U_k U_k \rangle}{3}\frac{\partial^2 f}{\partial U_j \partial U_j}\right] \tag{4}$$

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* Professor of Fluid Mechanics. Member AIAA.

$$u_j \frac{\partial z_\alpha f}{\partial x_j} = \frac{\langle U_k U_k \rangle^{1/2}}{2\lambda} \left(1 + \frac{L}{\langle u \rangle_\infty} \frac{\partial \langle u \rangle}{\partial y}\right) \times \left[2 \frac{\partial}{\partial U_j} (z_\alpha U_j f) + \frac{\langle U_k U_k \rangle}{3} \frac{\partial^2 z_\alpha f}{\partial U_j \partial U_j}\right] - \frac{\langle U_k U_k \rangle^{1/2}}{2\lambda} \times \left(1 + \frac{L}{\langle u \rangle_\infty} \frac{\partial \langle u \rangle}{\partial y}\right)(z_\alpha - \langle z_\alpha \rangle) f + W_\alpha f \quad (5)$$

where f is the distribution function of the fluid elements, and z_α is the concentration of the scalar quantity, such as the chemical species and heat, per unit mass of the fluid element. All subscripts in Eqs. (4) and (5), except α and ∞, are the Cartesian tensor indices. The subscript ∞ denotes the reference quantity. It is assumed that the flow is incompressible in spite of the fact that combustion may take place.

Equation (4), governing the distribution function f, has been solved in Ref. 5 for the present flowfield. Equation (5) represents five equations for the fuel, oxidant, combustion product, chemically inert species, and the temperature, respectively. These five equations are solved in the present study by the moment method employed earlier[5] to solve a similar equation.

Results and Discussion

It was found that the mean chemical state of the turbulent shear field is governed by the particular Damkohler number which is defined in terms of the ratio of the characteristic relaxation time of the energy-containing eddies to that of the molecular chemical reaction.

The typical structure of the diffusion flame obtained from the present analysis is shown in Figs. 1 and 2. In these figures, $\epsilon = 0$ and $\lambda/L = 1$ signify the infinitely large Damkohler number and the pure shear-produced turbulence.

Figure 1 shows the mean dimensionless concentration profiles of fuel, $\langle m \rangle$, oxidant, $\langle n \rangle$, and the combustion product $\langle z_p \rangle$. The mean dimensionless temperature profile, $\langle h \rangle$, is also shown for the three different temperature boundary conditions. Figure 2 shows the dimensionless mean transport rates of the fuel, $\langle MV \rangle$, oxidant, $\langle NV \rangle$, combustion product, $\langle Z_p V \rangle$, and the heat, $\langle HV \rangle$.

First, we see that the diffusion flame is very thick indeed in contrast to the infinitesimally thin flame sheet predicted by the existing theories.[1] In fact, the present analysis shows that the thickness of the flame zone is of the order of the local integral scale of turbulence which, in a pure shear-produced turbulence, is of the order of the shear layer thickness. A qualitative but clear agreement of the present findings, Fig. 1, with the experimental results[2-4] is seen.

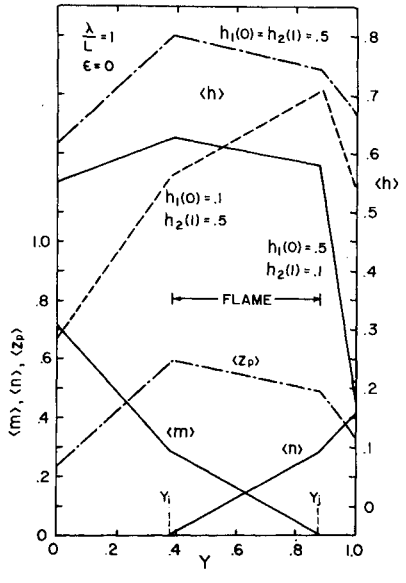

Fig. 1 Structure of equilibrium flame. $m_1(0) = 1$, $n_2(1) = 0.7$, $z_{c2}(1) = 0.3$, $d\phi/dY = 1$, $(a\hat{M}_r/d\hat{M}_p) = (b\hat{M}_f/d\hat{M}_p) = 1$.

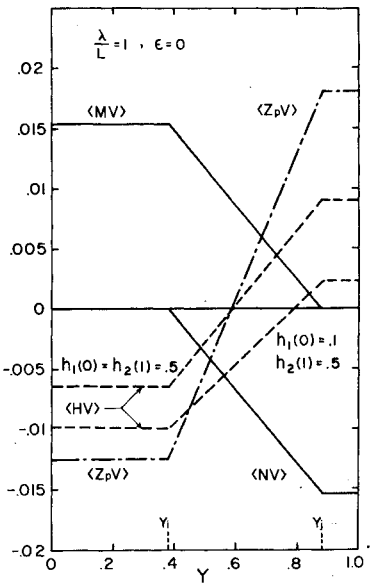

Fig. 2 Structure of equilibrium flame. $m_1(0) = 1$, $n_2(1) = 0.7$, $z_{c2}(1) = 0.3$, $d\phi/dY = 1$, $(a\hat{M}_r/d\hat{M}_p) = (b\hat{M}_f/d\hat{M}_p) = 1$.

A study of the two figures together shows that the heat transfer takes place in the direction of the increasing mean temperature in certain regions of the flame. A similar transport against the mean gradient is also evident for the combustion product in certain regions of the flame zone. Figures 1 and 2 show that the singularities in the mean gradients exist at the flame edges when the Damkohler number is infinitely large. As the Damkohler number becomes finite, these singularities are removed as the small nonequilibrium regions appear about the flame edges.[8] Therefore, the initial effect of decreasing Damkohler number is to make the flame edges less distinct rather than to vary the flame thickness.

Figure 1 shows that the temperature at either flame edge can be much greater than that at the other. Since the Damkohler number at the lower temperature edge can be much smaller, the singularity there will be smoothed out much earlier than that at the other edge. Under such circumstances, one would observe a pronounced temperature peak at one of the flame edges. This peak, however, should not be interpreted as the existence of a thin flame sheet. The detailed analyses as well as the complete results given in the preprint[8] will be published at a later date.

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Fluid Dynamics in a Large Arterial Bifurcation

Czeslaw M. Rodkiewicz* and Darrel H. Howell†
University of Alberta, Edmonton, Canada

Nomenclature

d = unstressed internal diameter of side branch
D = unstressed internal diameter of main branch
L = approximate length of entry region
Q_d = flow rate in side branch
Q_D = flow rate in main branch
Re = Reynolds number, Eq. (1)
R_o = unstressed internal radius of main branch
t = time variable
u = longitudinal coordinate of velocity
U = average longitudinal velocity
U' = fluctuating component of longitudinal velocity
α = unsteadiness parameter, Eq. (2)
β = ratio of branch diameters
γ = mass-flow ratio
θ = angle of branching
λ = normalized velocity fluctuation, Eq. (3)
ν = kinematic fluid viscosity
ω = pulse rate

Introduction

THE present work on the blood flow in a large arterial bifurcation was suggested by N. R. Kuchar whose Ph.D. thesis[1] and recent paper on blood flow devices[2] gave the necessary theoretical background. The primary objective is to investigate the effect of varying the significant dimensionless parameters on the manner in which the flow divides at a bifurcation.

The heart produces a periodic or pulsatile flow on the arterial side of the circulatory system. The amplitude of the flow pulse is largest in the aorta and becomes gradually smaller as the system branches. The arterial vessels are subjected to higher pressure and pressure variation, and they are thicker and contain more elastin than the venous system. Despite the extra stength and elasticity of the arterial walls, it seems likely that a system under the continual wear and tear of a pulsatile flow would be subject to many disorders. Such is the case, arterial diseae being a very great problem. One of the disorders is the atherosclerosis.[3] It refers to the build up of fatty and/or fibrous plaques which occlude the vessel and destroy the elasticity of its wall.

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Index categories: Space Medicine (Including Weightlessness, Radiation Effects, Psychology, etc.); Spacecraft Habitability and Life Support Systems.
* Research Director.
† Graduate Student.

Experimental Range

Flow in an arterial bifurcation is described by five dimensionless parameters.[1,2] The first two are the Reynolds number and the unsteadiness parameter, respectively,

$$Re = UD/\nu \tag{1}$$

$$\alpha = D(\omega/\nu)^{1/2} \tag{2}$$

It is now necessary to determine the range of these parameters for the physiological case. To do this, the following data has been selected[4,5]: a) The relative viscosity of blood with respect to distilled water varies from 3.5–5.4; b) blood density varies from 1.048–1.066 gm/cm^3; c) the kinematic viscosity of distilled water equals 0.010027 cm^2/sec; d) for an average case, blood flows at a mean velocity of 32 cm/sec through an aorta of 2.1 cm diam; e) human pulse rate varies from 50–100 beats per min; f) the diameter of the human aorta varies from 1.5–2.1 cm. Substitution of these data into Eqs. (1) and (2) yields the following experimental ranges of study: $1300 < Re < 2040$, $6 < \alpha < 15$.

Attinger et al.[6] show that a Fourier series can be used to approximate periodic blood flow in the cardiovascular system. The first harmonic, a sine wave, has the highest amplitude. We therefore assume that flow from the heart can be approximated as a sinusoidal flow. The normalized velocity fluctuation λ is defined by the following expression

$$\lambda = U'/U \tag{3}$$

where U is the mean velocity and U' is the peak to peak amplitude of the fluctuating component of velocity. Kuchar and Scala[2] report the physiological range for λ to be $1 < \lambda < 2$.

This work is confined to the case of the side branch off a straight vessel as illustrated in Fig. 1. Consideration of this bifurcation geometry gives rise to two additional dimensionless parameters—the angle between the branches, θ, and the ratio of the branch diameters, $\beta = d/D$. The following ranges of experimental study have been chosen for θ and β: $30° \leq \theta \leq 90°$, $0.20 \leq \beta \leq 0.80$.

Apparatus

Figure 2 gives the schematic of the apparatus. The bifurcations were made of glass. The inlet and outlet to the bifurcation were surgical tubing suspended by twine from a dexion frame. The upstream was approximately 8 ft in length and as straight and horizontal as possible. Kuchar and Ostrach[1] calculated that a length ratio (R_o/L) of 0.193 was necessary to ensure the elimination of end effects. The length ratio of the apparatus was 0.0065. Therefore, the entrance length of the apparatus was more than sufficient for flow stability.

For ease of observation, the internal diameter of the main branch of the experimental model has 1.25 in. Weiting[7]

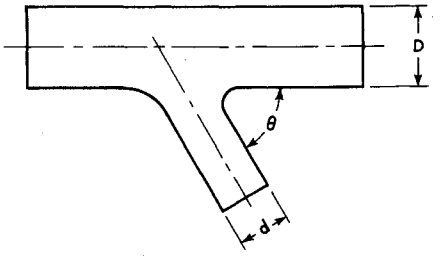

Fig. 1 The experimental bifurcation.